GENERAL HINGED SOLAR PANEL DYNAMICS APPROXIMATING FIRST-ORDER SPACECRAFT FLEXING

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Cody Allard[*], Hanspeter Schaub[†], and Scott Piggott[‡]

For many spacecraft with deployable structural components, such as solar panels or deployable antennas, the rigid-body assumption does not accurately model the full system dynamics. Spacecraft with large deployed solar panels exhibit flexible dynamics that can impact the final pointing and jitter performance of an attitude control system, or the simulation of an on-board accelerometer. For simulation and analysis purposes, it is desirable to include approximate flexible dynamics in a manner that easily integrates with the rigid body translational and rotational equations of motion. Current methods either require extensive derivation to implement flexible dynamics into the simulation or do not provide enough fidelity. This paper introduces a first-order model of the flexible dynamics using hinged multi-body dynamics that is applicable to a range of spacecraft shapes and configurations, but fully accounts for three-dimensional motion of this component. The formulation assumes the appended bodies are rigid bodies, and are connected to a main rigid body (hub) by a single degree of freedom torsional hinge. The numerical simulations are validated through a range of energy and momentum checks. A simple example of a simulation is included and highlights the necessity to include flexing for certain spacecraft.

INTRODUCTION

Spacecraft come in many shapes and sizes and some spacecraft have large appended solar panels or antennas. Typically these objects are connected to the spacecraft as cantilevered elements, therefore they are susceptible to flexing behavior. This behavior needs to be included in the dynamics. Often the spacecraft is assumed to be a rigid body, but this assumption will degrade the fidelity of the simulation if there are certain components that will flex. Flexing will impact both the translational and rotational motion (and associated stability margins) of the spacecraft, as well as sensor modeling such as accelerometers and rate gyros. For simulation and analysis purposes, flexing is very important because it can impact performance, requirements and success of the mission.

There are many different ways to model flexible dynamics. One method is to assume that the primary impact will be on the attitude dynamics of the spacecraft so the translational motion coupling can be ignored. Also, in some scenarios the effects of flexing can be assumed to only impact one plane of rotation, therefore one method is to constrain the motion to 1D rotational motion.[1] This approach allows the flexing body to be modeled as a finite number of masses on a cantilevered beam and allows for different frequency modes to be present.[1] This derivation results in a transfer

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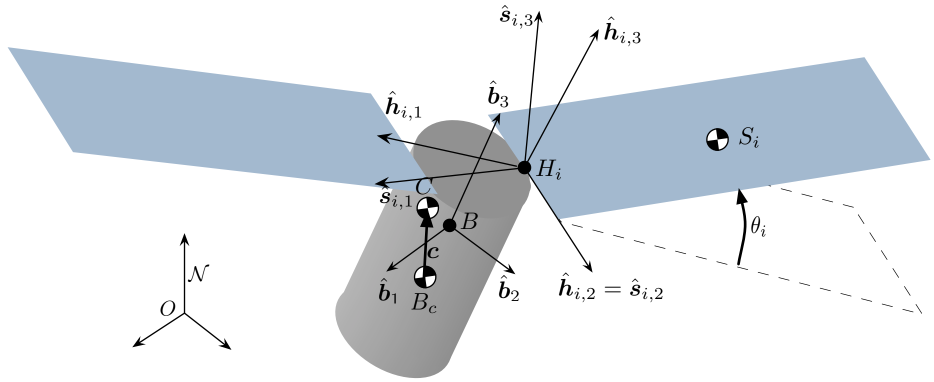


Figure 1. Components, variables and coordinate frames used for this derivation.

function that is useful in determining the stability and frequency response due to different inputs. However, it neglects the cross coupling affect on the other rotational axes, and the effect on translational motion. This method is helpful in the early stages of a mission, but lacks fidelity and is limited in its application.

In contrast, the field of multi-body dynamics has extensive research on modeling flexible dynamics and the equations of motion presented are generalized for complex and diverse problems. This results in requiring derivation of equations because of generality.[2–4] These methods are required for unique and complex systems because the equations of motion depend on how many joints that are interconnected. For example, in robotic systems, the number of interconnected joints varies widely, and the equations of motion are specific to that system.[5,6] Since there are many spacecraft that have similar designs with appended solar panels, there is a need to develop equations of motion that could be applied to these spacecraft.

Similar to this paper, multiple publications present models of spacecraft dynamics with appended solar panels.[7–9] However, this previous research is mainly focused on the deployment of solar panels and how the deployment affects the dynamics of the spacecraft.[7–9] Also, the previous research on deployable solar panels are specific to solar panels that are composed of interconnected bodies. This paper considers systems where the solar panels are single rigid bodies.

This paper introduces a method of modeling the flexible dynamics of the solar panels by assuming that the hub of the spacecraft and the solar panels are rigid bodies, but the solar panels are connected to the hub by single degree-of-freedom torsional springs. The torsional spring constants could be attenuated to match the natural frequencies of the solar panels which could be found from Finite Element Analysis or testing. This method in modeling the flexible dynamics is a first order model, and other effects like bending and torsional bending could be added later. However, in contrast to earlier work, the multi-body system is allowed to undergo general three-dimensional motions in translation and rotation.

PROBLEM STATEMENT

The purpose of this paper is to develop equations of motion describing flexible dynamics of a spacecraft that can be smoothly integrated into computer simulation. It will reduce the need of deriving equations of motion for new missions. This formulation is completed in a general way that applies to a wide range of spacecraft. The description of the spacecraft, componenets, coordinate frames and variables are introduced in Figure 1. This describes each component being considered in the derivation.

Figure 1 is composed of a rigid hub connected to two solar panels by one degree-of-freedom joints. These joints are composed of torsional hinges that have a linear spring constant of k_i, and an angular rate damping term, c_i. Two panels are shown for simplicity, however the formulation assumes there are N solar panels.

There are four coordinate frames defined for this formulation. The inertial reference frame is indicated by $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$. The body fixed coordinate frame, $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$, is defined with its origin, B, to be coincident with the center of mass of the spacecraft when the solar panels are undeflected and can be oriented in any direction. The solar panel frames, $\mathcal{S}_i : \{\hat{\boldsymbol{s}}_{i,1}, \hat{\boldsymbol{s}}_{i,2}, \hat{\boldsymbol{s}}_{i,3}\}$, are frames with their origins at the location of their hinge joints, H_i. The $\mathcal{S}_i$ frame is oriented such that $\hat{\boldsymbol{s}}_{i,1}$ points anti-parallel to the center of mass of the solar panel, S_i. The $\hat{\boldsymbol{s}}_{i,2}$ axis is defined as the rotation axis that would yield a positive θ_i using the right-hand rule. The hinge frames, $\mathcal{H}_i : \{\hat{\boldsymbol{h}}_{i,1}, \hat{\boldsymbol{h}}_{i,2}, \hat{\boldsymbol{h}}_{i,3}\}$, are frames fixed with respect to the body frame, and are equivalent to the respective $\mathcal{S}_i$ frames when the solar panel is undeflected. The i indicates the i^{th} solar panel.

Point B is the origin of the body frame, C is the center of mass location of the entire spacecraft, B_c is the center mass location of the the rigid hub, and S_i is the center of mass of the i^{th} solar panel. The vector $\boldsymbol{c}$ points from the origin of the body frame to the center of mass of the spacecraft. The formulation assumes that if there is no deflection in the solar panels, then point B will be coincident with point C. The variable d_i defines the distance between points H_i and S_i.

The remaining sections of the paper explain the derivation of the equations using Newtonian and Eulerian mechanics and give results of a tutorial simulation. Equations of motion (EOM) are required for the translational, rotational, and solar panel motion.

DERIVATION OF EQUATIONS OF MOTION

Spacecraft Translational Motion

The derivation begins with Newton's first law for the center of mass of the spacecraft.

$$\ddot{\boldsymbol{r}}_{C/N} = \frac{\boldsymbol{F}}{m_{\text{sc}}} \tag{1}$$

Ultimately the acceleration of the body frame or point B is desired

$$\ddot{\boldsymbol{r}}_{B/N} = \ddot{\boldsymbol{r}}_{C/N} - \ddot{\boldsymbol{c}} \tag{2}$$

The definition of $\boldsymbol{c}$ can be seen in Eq. (3).

$$\boldsymbol{c} = \frac{m_{\text{hub}}\boldsymbol{r}_{B_c/B} + \sum_i^N m_{\text{sp}_i}\boldsymbol{r}_{S_i/B}}{m_{\text{sc}}} \tag{3}$$

To find the inertial time derivative of $\boldsymbol{c}$, it is first necessary to find the time derivative of $\boldsymbol{c}$ with respect to the body frame. A time derivative of any vector, $\boldsymbol{v}$, with respect to the body frame is denoted by $\boldsymbol{v}'$; the inertial time derivative is labeled as $\dot{\boldsymbol{v}}$. The first and second body-relative time derivatives of $\boldsymbol{c}$ can be seen in Eqs. (4) and (5).

$$\boldsymbol{c}' = \frac{\sum_i^N m_{\text{sp}_i}\boldsymbol{r}'_{S_i/H_i}}{m_{\text{sc}}} \tag{4}$$

$$\boldsymbol{c}'' = \frac{\sum_i^N m_{\text{sp}_i} \boldsymbol{r}''_{S_i/H_i}}{m_{\text{sc}}} \tag{5}$$

The variable $\boldsymbol{r}_{S_i/Hi}$ can easily be defined using the $\hat{\boldsymbol{s}}_{i,1}$ axis

$$\boldsymbol{r}_{S_i/H_i} = -d_i \hat{\boldsymbol{s}}_{i,1} \tag{6}$$

Now the first and second time derivatives with respect to the body frame of $\boldsymbol{r}_{S_i/H_i}$ are taken

$$\boldsymbol{r}'_{S_i/H_i} = d_i \dot{\theta}_i \hat{\boldsymbol{s}}_{i,3} \tag{7}$$

$$\boldsymbol{r}''_{S_i/H_i} = d_i \left(\ddot{\theta}_i \hat{\boldsymbol{s}}_{i,3} + \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} \right) \tag{8}$$

Eqs. (4) and (5) can now include these new definitions and yields

$$\boldsymbol{c}' = \frac{\sum_i^N m_{\text{sp}_i} d_i \dot{\theta}_i \hat{\boldsymbol{s}}_{i,3}}{m_{\text{sc}}} \tag{9}$$

$$\boldsymbol{c}'' = \frac{\sum_i^N m_{\text{sp}_i} d_i \left(\ddot{\theta}_i \hat{\boldsymbol{s}}_{i,3} + \dot{\theta}_i^2 \hat{\boldsymbol{s}}_{i,1} \right)}{m_{\text{sc}}} \tag{10}$$

Using the transport theorem[10] yields the following definition for $\ddot{\boldsymbol{c}}$

$$\ddot{\boldsymbol{c}} = \boldsymbol{c}'' + 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}' + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c} \right) \tag{11}$$

Eq. (2) can be updated to include Eq. (11)

$$\ddot{\boldsymbol{r}}_{B/N} = \ddot{\boldsymbol{r}}_{C/N} - \boldsymbol{c}'' - 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}' - \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c} \right) \tag{12}$$

Eq. (12) is one of the three equations required to describe the motion of the spacecraft. It describes the motion of the body frame with respect to the inertial frame and is in terms of the rotational motion and solar panel motion. In the next section, the EOM for a solar panel is derived.

Solar Panel Motion

The solar panel frame $\mathcal{S}_i$ is assumed to be a principle frame such that the solar panel inertia tensor about its center of mass is

$$[I_{S_i}] = {}^{\mathcal{S}}\begin{bmatrix} I_{s_{i,1}} & 0 & 0 \\ 0 & I_{s_{i,2}} & 0 \\ 0 & 0 & I_{s_{i,3}} \end{bmatrix} \tag{13}$$

Let $\boldsymbol{L}_{H_i} = L_{i,1}\hat{\boldsymbol{s}}_{i,1} + L_{i,2}\hat{\boldsymbol{s}}_{i,2} + L_{i,3}\hat{\boldsymbol{s}}_{i,3}$ be the total torque acting on the solar panel. The corresponding hinge torque is given through

$$L_{i,2} = -k_i \theta_i - c_i \dot{\theta}_i \tag{14}$$

The hinge structure produces the other two torques $L_{i,1}$ and $L_{i,3}$.

The inertial angular velocity vector for the solar panel frame is

$$\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{H}_i} + \boldsymbol{\omega}_{\mathcal{H}_i/\mathcal{B}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \tag{15}$$

where $\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{H}_i} = \dot{\theta}_i \hat{\boldsymbol{s}}_{i,2}$. Because the hinge frame $\mathcal{H}_i$ is fixed relative to the body frame $\mathcal{B}$ the relative angular velocity vector is $\boldsymbol{\omega}_{\mathcal{H}_i/\mathcal{B}} = \boldsymbol{0}$. The body angular velocity vector is written in $\mathcal{S}_i$-frame components as

$$\begin{aligned}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} &= (\hat{\boldsymbol{s}}_{i,1} \cdot \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}})\hat{\boldsymbol{s}}_{i,1} + (\hat{\boldsymbol{s}}_{i,2} \cdot \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}})\hat{\boldsymbol{s}}_{i,2} + (\hat{\boldsymbol{s}}_{i,3} \cdot \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}})\hat{\boldsymbol{s}}_{i,3} && (16)\\ &= \omega_{s_{i,1}}\hat{\boldsymbol{s}}_{i,1} + \omega_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + \omega_{s_{i,3}}\hat{\boldsymbol{s}}_{i,3} && (17)\end{aligned}$$

Using this definition greatly simplifies the following algebraic development. Finally, the inertial solar panel angular velocity vector is written as

$$\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} = \omega_{s_{i,1}}\hat{\boldsymbol{s}}_{i,1} + (\omega_{s_{i,2}} + \dot{\theta}_i)\hat{\boldsymbol{s}}_{i,2} + \omega_{s_{i,3}}\hat{\boldsymbol{s}}_{i,3} \tag{18}$$

As $\hat{\boldsymbol{s}}_{i,2}$ is a body-fixed vector, note that

$$\dot{\omega}_{s_{i,2}} = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \cdot \hat{\boldsymbol{s}}_{i,2}\right) = \frac{{}^{\mathcal{B}}\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}\right) \cdot \hat{\boldsymbol{s}}_{i,2} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \cdot \hat{\boldsymbol{s}}_{i,2} \tag{19}$$

Substituting these angular velocity components into the rotational equations of motion of a rigid body with torques taken about its center of mass,[10] the general solar panel equations of motion are written as

$$\begin{aligned}I_{s_{i,1}}\dot{\omega}_{s_{i,1}} &= -(I_{s_{i,3}} - I_{s_{i,2}})(\omega_{s_{i,2}} + \dot{\theta}_i)\omega_{s_{i,3}} + L_{s_{i,1}} && (20)\\ I_{s_{i,2}}(\dot{\omega}_{s_{i,2}} + \ddot{\theta}_i) &= -(I_{s_{i,1}} - I_{s_{i,3}})\omega_{s_{i,3}}\omega_{s_{i,1}} + L_{s_{i,2}} && (21)\\ I_{s_{i,3}}\dot{\omega}_{s_{i,3}} &= -(I_{s_{i,2}} - I_{s_{i,1}})\omega_{s_{i,1}}(\omega_{s_{i,2}} + \dot{\theta}_i) + L_{s_{i,3}} && (22)\end{aligned}$$

where $\boldsymbol{L}_{S_i} = L_{s_{i,1}}\hat{\boldsymbol{s}}_{i,1} + L_{s_{i,2}}\hat{\boldsymbol{s}}_{i,2} + L_{s_{i,3}}\hat{\boldsymbol{s}}_{i,3}$ is the net torque acting on the solar panel about its center of mass. The second differential equation is used to get the equations of motion of θ_i. The first and third equation could used to back-solve for the structural hinge torques embedded in $L_{s_{i,1}}$ and $L_{s_{i,3}}$ if needed.

Let $\boldsymbol{F}_{S_i}$ be the net force acting on the solar panel. Using the superparticle theorem[10] yields

$$\boldsymbol{F}_{S_i} = m_{\text{sp}_i}\ddot{\boldsymbol{r}}_{S_i/N} \tag{23}$$

The torque about the solar panel center of mass can be related to the torque about the hinge point H_i using

$$\boldsymbol{L}_{H_i} = \boldsymbol{L}_{S_i} + \boldsymbol{r}_{S_i/H_i} \times \boldsymbol{F}_{S_i} \tag{24}$$

Solving for the torque about S_i yields

$$\boldsymbol{L}_{S_i} = \boldsymbol{L}_{H_i} - \boldsymbol{r}_{S_i/H_i} \times m_{\text{sp}_i}\ddot{\boldsymbol{r}}_{S_i/N} \tag{25}$$

Taking the vector dot product with $\hat{\boldsymbol{s}}_{i,2}$ and using $\boldsymbol{r}_{S_i/H_i} = -d_i\hat{\boldsymbol{s}}_{i,1}$ results in

$$\begin{aligned}L_{s_{i,2}} &= \hat{\boldsymbol{s}}_{i,2} \cdot \boldsymbol{L}_{S_i} = \underbrace{\hat{\boldsymbol{s}}_{i,2} \cdot \boldsymbol{L}_{H_i}}_{L_{i,2}} - \hat{\boldsymbol{s}}_{i,2} \cdot \left(\boldsymbol{r}_{S_i/H_i} \times m_{\text{sp}_i}\ddot{\boldsymbol{r}}_{S_i/N}\right) && (26)\\ &= -k_i\theta - c_i\dot{\theta}_i + m_{\text{sp}_i}d_i\hat{\boldsymbol{s}}_{i,2} \cdot \left(\hat{\boldsymbol{s}}_{i,1} \times \ddot{\boldsymbol{r}}_{S_i/N}\right) && (27)\end{aligned}$$

Taking two inertial time derivatives of $\boldsymbol{r}_{S_i/N} = \boldsymbol{r}_{H_i/N} - d\hat{\boldsymbol{s}}_{i,1}$ yields

$$\ddot{\boldsymbol{r}}_{S_i/N} = \ddot{\boldsymbol{r}}_{H_i/N} - \dot{\boldsymbol{\omega}}_{\mathcal{S}_i/\mathcal{N}} \times (d\hat{\boldsymbol{s}}_{i,1}) - \boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} \times (d\hat{\boldsymbol{s}}_{i,1})) \tag{28}$$

Substituting this inertial acceleration into the above L_{s_2} expression provides

$$L_{s_{i,2}} = -k_i\theta_i - c_i\dot{\theta}_i + m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,2} \cdot (\hat{\boldsymbol{s}}_{i,1} \times \ddot{\boldsymbol{r}}_{H_i/N}) + m_{\text{sp}_i} d_i^2 \hat{\boldsymbol{s}}_{i,2} \cdot (\hat{\boldsymbol{s}}_{i,1} \times (\hat{\boldsymbol{s}}_{i,1} \times \dot{\boldsymbol{\omega}}_{\mathcal{S}_i/\mathcal{N}})) - m_{\text{sp}_i} d_i^2 \hat{\boldsymbol{s}}_{i,2} \cdot (\hat{\boldsymbol{s}}_{i,1} \times (\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{S}_i/\mathcal{N}} \times \hat{\boldsymbol{s}}_{i,1}))) \quad (29)$$

Using the double vector cross product identity, as well as $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$, the $L_{s_{i,2}}$ torque component is simplified to

$$L_{s_{i,2}} = -k_i\theta_i - c_i\dot{\theta}_i - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3} \cdot \ddot{\boldsymbol{r}}_{H_i/N} - m_{\text{sp}_i} d_i^2 \hat{\boldsymbol{s}}_{i,2} \cdot \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - m_{\text{sp}_i} d_i^2 \ddot{\theta}_i + m_{\text{sp}_i} d_i^2 \omega_{s_{i,3}} \omega_{s_{i,1}} \quad (30)$$

Substituting this torque into the earlier differential equation

$$I_{s_{i,2}}(\dot{\omega}_{s_{i,2}} + \ddot{\theta}_i) = -(I_{s_{i,1}} - I_{s_{i,3}})\omega_{s_{i,3}}\omega_{s_{i,1}} + L_{s_{i,2}} \quad (31)$$

leads to the desired scalar hinged solar panel equation of motion

$$\left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right) \hat{\boldsymbol{s}}_{i,2}^T \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right) \ddot{\theta}_i + m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \ddot{\boldsymbol{r}}_{H_i/N} + k_i\theta + c_i\dot{\theta}_i + \left(I_{s_{i,1}} - I_{s_{i,3}} - m_{\text{sp}_i} d_i^2\right) \omega_{s_{i,3}}\omega_{s_{i,1}} = 0 \quad (32)$$

The term $\ddot{\boldsymbol{r}}_{H_i/N}$ needs to be expanded to be in terms of the desired translational motion $\ddot{\boldsymbol{r}}_{B/N}$. Knowing that the hinge location is a fixed point on the body, Eq. (32) is changed to the following form

$$\left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right) \hat{\boldsymbol{s}}_{i,2}^T \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T (\ddot{\boldsymbol{r}}_{B/N} + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{H_i/N} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{H_i/N})) + \left(I_{s_{i,2}} + m_{\text{sp}_i} d_i^2\right) \ddot{\theta}_i + k_i\theta_i + c_i\dot{\theta}_i + \left(I_{s_{i,1}} - I_{s_{i,3}} - m_{\text{sp}_i} d_i^2\right) \omega_{s_{i,3}}\omega_{s_{i,1}} = 0 \quad (33)$$

Eq. (33) is the second EOM required to describe the motion of the spacecraft and will be utilized later in the paper. The next section explains the formulation of the rotational motion.

Spacecraft Rotational Motion

The last EOM that needs to be developed is the rotational motion. Starting with Euler's equation when the body fixed coordinate frame origin is not coincident with the center of mass of the body[10]

$$\dot{\boldsymbol{H}}_{\text{sc},B} = \boldsymbol{L}_B + m_{\text{sc}} \ddot{\boldsymbol{r}}_{B/N} \times \boldsymbol{c} \quad (34)$$

where $\boldsymbol{L}_B$ is the total external torque about point B. The definition of the angular momentum vector of the spacecraft about point B is:

$$\boldsymbol{H}_{\text{sc},B} = [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}} \boldsymbol{r}_{B_c/B} \times \dot{\boldsymbol{r}}_{B_c/B} + \sum_i^N \left([I_{\text{sp}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \dot{\theta}_i I_{\text{sp}_i,S_i,2} \hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} \boldsymbol{r}_{S_i/B} \times \dot{\boldsymbol{r}}_{S_i/B}\right) \quad (35)$$

Now the inertial derivative of Eq. (35) is taken and yields

$$\dot{\boldsymbol{H}}_B = [I_{\text{hub},B_c}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}} \boldsymbol{r}_{B_c/B} \times \ddot{\boldsymbol{r}}_{B_c/B} + \sum_i^N \Big([I'_{\text{sp}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [I_{\text{sp}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{sp}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + \ddot{\theta}_i I_{\text{sp}_i,S_i,2} \hat{\boldsymbol{h}}_{i,2} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \dot{\theta}_i I_{\text{sp}_i,S_i,2} \hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i} \boldsymbol{r}_{S_i/B} \times \ddot{\boldsymbol{r}}_{S_i/B}\Big) \quad (36)$$

The terms $\ddot{\boldsymbol{r}}_{B_c/B}$ and $\ddot{\boldsymbol{r}}_{S_i/B}$ are found using the transport theorem and knowing that $\boldsymbol{r}_{B_c/B}$ is fixed with respect to the body frame.

$$\ddot{\boldsymbol{r}}_{B_c/B} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{B_c/B} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{B_c/B}) \tag{37}$$

$$\ddot{\boldsymbol{r}}_{S_i/B} = \boldsymbol{r}''_{S_i/B} + 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}'_{S_i/B} + \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}) \tag{38}$$

Incorporating Eqs. (37) and (38) into Eq. (36) results in:

$$\begin{aligned}\dot{\boldsymbol{H}}_B = {} & [I_{\text{hub},B_c}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{hub},B_c}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + m_{\text{hub}}\boldsymbol{r}_{B_c/B} \times (\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{B_c/B}) \\ & + m_{\text{hub}}\boldsymbol{r}_{B_c/B} \times [\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{B_c/B})] + \sum_i^N \Big([I'_{\text{sp}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [I_{\text{sp}_i,S_i}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{sp}_i,S_i}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \\ & + \ddot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \dot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times \boldsymbol{r}''_{S_i/B} + 2m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}'_{S_i/B}) \\ & + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times (\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B}) + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times [\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{S_i/B})] \Big)\end{aligned} \tag{39}$$

Introducing the tilde operator to replace the cross product, and applying the parallel axis theorem the following can be defined:

$$[I_{\text{hub},B}] = [I_{\text{hub},B_c}] + m_{\text{hub}}[\tilde{\boldsymbol{r}}_{B_c/B}][\tilde{\boldsymbol{r}}_{B_c/B}]^T \tag{40}$$

$$[I_{\text{sp}_i,B}] = [I_{\text{sp}_i,S_i}] + m_{\text{sp}_i}[\tilde{\boldsymbol{r}}_{S_i/B}][\tilde{\boldsymbol{r}}_{S_i/B}]^T \tag{41}$$

$$[I_{\text{sc},B}] = [I_{\text{hub},B}] + \sum_i^N [I_{\text{sp}_i,B}] \tag{42}$$

$$[I'_{\text{sc},B}] = \sum_i^N \Big\{ \Big([I'_{\text{sp}_i,S_i}] - 2m_{\text{sp}_i}[\tilde{\boldsymbol{r}}_{S_i/B}][\tilde{\boldsymbol{r}}'_{S_i/B}] \Big) \tag{43}$$

This produces

$$\begin{aligned}\dot{\boldsymbol{H}}_B = {} & [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \\ & + \sum_i^N \Big\{ \ddot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \dot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times \boldsymbol{r}''_{S_i/B} \Big\}\end{aligned} \tag{44}$$

Eqs. (34) and (44) are equated and yield

$$\begin{aligned}\boldsymbol{L}_B + m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} \times \boldsymbol{c} = {} & [I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times [I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \\ & + \sum_i^N \Big\{ \ddot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \dot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times \boldsymbol{r}''_{S_i/B} \Big\}\end{aligned} \tag{45}$$

Finally, using skew symmetric matrix and simplifying yields the modified Euler equation, which is the last EOM necessary to describe the motion of the spacecraft.

$$\begin{aligned}[I_{\text{sc},B}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = {} & -[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \sum_i^N \Big\{ \ddot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} \\ & + [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\dot{\theta}_i I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}\boldsymbol{r}_{S_i/B} \times \boldsymbol{r}''_{S_i/B} \Big\} + \boldsymbol{L}_B + m_{\text{sc}}\ddot{\boldsymbol{r}}_{B/N} \times \boldsymbol{c}\end{aligned} \tag{46}$$

SIMULATION IMPLEMENTATION

The equations presented in the previous sections result in $N+6$ coupled differential equations. Therefore if the EOM were placed into state space form, a system mass matrix of size $N+6$ would need to be inverted to numerically integrate the EOM. This can result in a computationally expensive simulation. In the following section, the EOM are manipulated to increase the efficiency. This manipulation involves inverting an $N \times N$ matrix for the solar panel motion, inverting the rotational motion equation (3×3), and then back solving for the solar panel and translational motions. This derivation can be seen in the following sections.

Solar Panel Motion

In Eq. (33), the solar panel motion is coupled with the translational motion and the rotational motion. The translational motion needs to be decoupled from the solar panel motion. To perform this task, Eq. (12) is substituted into Eq. (32). After some simplification this substitution yields

$$\left[\left(I_{sp_{i,2}} + m_{\text{sp}_i} d_i^2\right) - \frac{m_{\text{sp}_i}^2}{m_{\text{sc}}} d_i^2\right] \ddot{\theta}_i - \frac{m_{\text{sp}_i}}{m_{\text{sc}}} d_i \hat{\boldsymbol{s}}_{i,3}^T \left(\sum_{j=1; j\neq i}^{N} m_{sp_j} d_j \ddot{\theta}_j \hat{\boldsymbol{s}}_{j,3}\right) =$$
$$- \left[\left(I_{sp_{i,2}} + m_{\text{sp}_i} d_i^2\right) \hat{\boldsymbol{s}}_{i,2}^T + m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \left([\tilde{\boldsymbol{c}}] - [\tilde{\boldsymbol{r}}_{H_i/B}]\right)\right] \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - k_i \theta - c_i \dot{\theta}_i$$
$$+ \left(I_{sp_{i,3}} - I_{sp_{i,1}} - + m_{\text{sp}_i} d_i^2\right) \omega_{s_{i,3}} \omega_{s_{i,1}} - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \Big[\ddot{\boldsymbol{r}}_{C/N} - 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}' - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}\right)$$
$$+ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{H_i/N}\right) - \frac{1}{m_{\text{sc}}} \left(\sum_{j=1; j\neq i}^{N} m_{sp_j} d_j \dot{\theta}_j^2 \hat{\boldsymbol{s}}_{j,1}\right)\Big] \tag{47}$$

Eq. (47) is written in matrix form to utilize some linear algebra techniques.

$$[A] \begin{bmatrix} \ddot{\theta}_1 \\ \cdot \\ \cdot \\ \ddot{\theta}_N \end{bmatrix} = [F] \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{P} \tag{48}$$

Where $[A]$ is an $N \times N$ matrix with the following definitions

$$a_{i,i} = \left[\left(I_{sp_{i,2}} + m_{\text{sp}_i} d_i^2\right) - \frac{m_{\text{sp}_i}^2}{m_{\text{sc}}} d_i^2\right] \tag{49a}$$

$$a_{i,j} = -\frac{m_{\text{sp}_i}}{m_{\text{sc}}} d_i \hat{\boldsymbol{s}}_{i,3}^T \left(\sum_{j=1; j\neq i}^{N} m_{sp_j} d_j \ddot{\theta}_j \hat{\boldsymbol{s}}_{j,3}\right) \tag{49b}$$

$[F]$ is an $N \times 3$ matrix with its row elements defined as

$$\boldsymbol{f}_i^T = - \left[\left(I_{sp_{i,2}} + m_{\text{sp}_i} d_i^2\right) \hat{\boldsymbol{s}}_{i,2}^T + m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \left([\tilde{\boldsymbol{c}}] - [\tilde{\boldsymbol{r}}_{H_i/B}]\right)\right] \tag{50}$$

$\boldsymbol{P}$ is an $N \times 1$ vector with the following elements

$$p_i = -k_i \theta - c_i \dot{\theta}_i + \left(I_{sp_{i,3}} - I_{sp_{i,1}} - + m_{\text{sp}_i} d_i^2\right) \omega_{s_{i,3}} \omega_{s_{i,1}} - m_{\text{sp}_i} d_i \hat{\boldsymbol{s}}_{i,3}^T \Big[\ddot{\boldsymbol{r}}_{C/N} - 2\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}'$$
$$- \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{c}\right) + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \left(\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{r}_{H_i/N}\right) - \frac{1}{m_{\text{sc}}} \left(\sum_{j=1; j\neq i}^{N} m_{sp_j} d_j \dot{\theta}_j^2 \hat{\boldsymbol{s}}_{j,1}\right)\Big] \tag{51}$$

Eq. (48) can now be solved by inverting matrix $[A]$ ($[E] = [A]^{-1}$).

$$\begin{bmatrix}\ddot{\theta}_1 & . & . & \ddot{\theta}_N\end{bmatrix}^T = [E][F]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + [E]\boldsymbol{P} \tag{52}$$

Since the modified Euler's equation, Eq. (46), has $\ddot{\theta}_i$ terms, it is more convenient to use the expression for $\ddot{\theta}_i$ as

$$\ddot{\theta}_i = \boldsymbol{e}_i^T[F]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \boldsymbol{e}_i^T\boldsymbol{P} \tag{53}$$

Where the subcomponents of $[E]$ are defined as

$$[E] = \begin{bmatrix}\boldsymbol{e}_1^T\\ .\\ .\\ \boldsymbol{e}_N^T\end{bmatrix} \tag{54}$$

Rotational Motion

The rotational motion is coupled with both the solar panel and translational motion. Therefore, the translational and solar panel EOM need to be substituted into the modified Euler's equation, Eq. (46). This substitution and simplification is performed and results in

$$\begin{aligned}\left([I_{\text{sc},B}] + m_{\text{sc}}[\tilde{\boldsymbol{c}}][\tilde{\boldsymbol{c}}]\right)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} &+ \sum_i^N \left[I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}d_i\left([\tilde{\boldsymbol{r}}_{S_i/B}] - [\tilde{\boldsymbol{c}}]\right)\hat{\boldsymbol{s}}_{i,3}\right]\ddot{\theta}_i\\ &= \boldsymbol{L}_B - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}\\ &- m_{\text{sc}}[\tilde{\boldsymbol{c}}]\left(\ddot{\boldsymbol{r}}_{C/N} - 2[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}\right)\\ &- \sum_i^N \left\{I_{\text{sp}_i,S_i,2}\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}d_i\dot{\theta}_i^2\left([\tilde{\boldsymbol{r}}_{S_i/B}] - [\tilde{\boldsymbol{c}}]\right)\hat{\boldsymbol{s}}_{i,1}\right\}\end{aligned} \tag{55}$$

For simplification purposes, a new set of variables are defined and Eq. (55) simplifies to

$$[I_{\text{sc},C}]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} + \sum_i^N \boldsymbol{R}_i\ddot{\theta}_i = \boldsymbol{S} \tag{56}$$

Where $\boldsymbol{R}_i$ and $\boldsymbol{S}$ are 3×1 vectors and are defined in the following equations

$$\boldsymbol{R}_i = \left[I_{\text{sp}_i,S_i,2}\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}d_i\left([\tilde{\boldsymbol{r}}_{S_i/B}] - [\tilde{\boldsymbol{c}}]\right)\hat{\boldsymbol{s}}_{i,3}\right] \tag{57}$$

$$\begin{aligned}\boldsymbol{S} = \boldsymbol{L}_B - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][I_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - [I'_{\text{sc},B}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - m_{\text{sc}}[\tilde{\boldsymbol{c}}]\left(\ddot{\boldsymbol{r}}_{C/N} - 2[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}' - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}][\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\boldsymbol{c}\right)\\ - \sum_i^N \left\{I_{\text{sp}_i,S_i,2}\dot{\theta}_i[\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}]\hat{\boldsymbol{h}}_{i,2} + m_{\text{sp}_i}d_i\dot{\theta}_i^2\left([\tilde{\boldsymbol{r}}_{S_i/B}] - [\tilde{\boldsymbol{c}}]\right)\hat{\boldsymbol{s}}_{i,1}\right\}\end{aligned} \tag{58}$$

Substituting Eq. (53) into Eq. (56) results in

$$\left([I_{\text{sc},C}] + \sum_i^N \boldsymbol{R}_i\boldsymbol{e}_i^T[F]\right)\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{S} - \sum_i^N \boldsymbol{R}_i[E]\boldsymbol{P} \tag{59}$$

$[Z]$ and $\boldsymbol{V}$ are new variables defined in the following equations

$$[Z] = [I_{\text{sc},C}] + \sum_{i}^{N} \boldsymbol{R}_i \boldsymbol{e}_i^T [F] \tag{60}$$

$$\boldsymbol{V} = \boldsymbol{S} - \sum_{i}^{N} \boldsymbol{R}_i [E] \boldsymbol{P} \tag{61}$$

$[Z]$ is a 3×3 matrix and $\boldsymbol{V}$ is a 3×1 vector. This allows the final equation to be simplified to the following form

$$[Z]\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} = \boldsymbol{V} \tag{62}$$

Now Eq. (62) can be solved for $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$.

It is important to note that there are two remaining steps required to implement these equations into a simulation. The solution for $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ is substituted into the solar panel motion equation, Eq. (53) to solve for $\ddot{\theta}_i$. And finally, the solution for $\ddot{\theta}_i$ and $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ is placed into the translational motion equation, Eq. (12). This concludes the necessary steps needed to implement flexible dynamics into a computer simulation. The recommended coordinate frames for this simulation are to solve everything in the body frame, $\mathcal{B}$, and then before integration, place the translational motion in the inertial frame, $\mathcal{N}$. However, this formulation is general, and any coordinate frames can be chosen as needed.

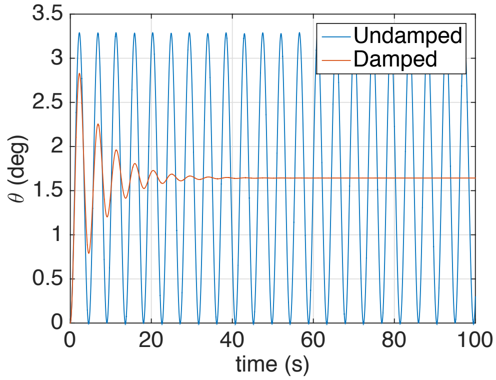
This simplification results in a much more efficient simulation. The simplified EOM takes 62% of the time it take the original formulation to execute for two solar panels. This is a dramatic speed up and is very desirable.

NUMERICAL SIMULATION

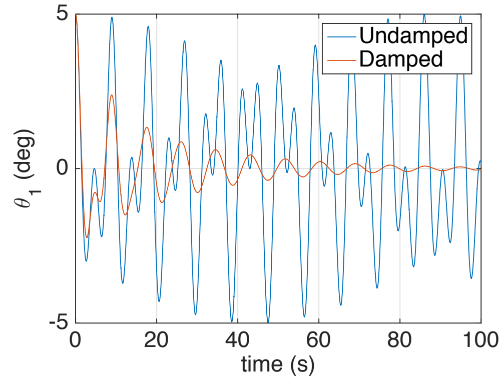
To validate the EOM and to provide a simple example of the flexing behavior, the spacecraft shown in Figure 1 is used. The hub is a cylinder with its center of mass located at the center of the cylinder. It has two identical solar panels modeled as rectangular prisms located opposite from each other. There are two scenarios: one with damping and one without.

For simplicity, the spacecraft is given initial conditions that will constrain the spacecraft to planar motion. This is done by aligning the inertial frame and body frame initially, having no external forces acting on the body, and giving the solar panel deflection, θ_1, an initial value of 5°. All other initial values are set to zero. A new angle, θ is defined and is the angle between the $\hat{\boldsymbol{b}}_3$ and $\hat{\boldsymbol{n}}_3$. It should be noted here that θ is only defined for use in this planar case, but otherwise 3-dimensional orientation descriptions would need to be used to describe the relationship between the $\mathcal{B}$ and $\mathcal{N}$ frames. The results from this simulation can be seen in Figures 2-4.

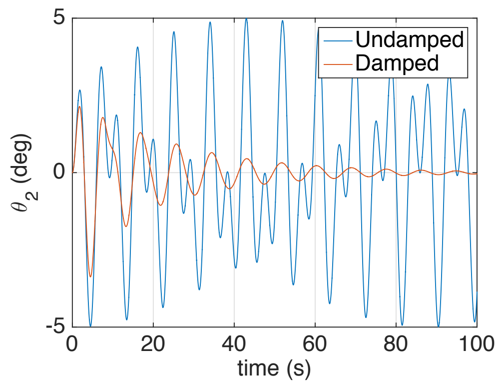
Knowing that total energy needs to be conserved for the undamped case and momentum needs to be conserved for both cases, the results agree with this insight. The first solar panel initially responds by traveling back to equilibrium, and the rotational, translational motions are each affected. Fig. 4 shows that momentum is conserved in both cases, energy is conserved in the undamped case, and energy decays to zero in the damped scenario. This gives confidence in the formulation presented.



(a) Orientation of body frame, θ



(b) Solar panel 1 angle, θ_1



(c) Solar panel 2 angle, θ_2

Figure 2. Rotational and solar panel motion

CONCLUSION

This paper presents a very convenient and compact formulation for a first-order approximation for flexible dynamics that can be applied to spacecraft with appended solar panels or hinged structural sub-components that can be modeled as single rigid bodies. It is not applicable to appended solar panels that consist of multiple interconnect panels. For applicable missions, the EOM would not need to be rederived and they could be smoothly integrated into simulation of the spacecraft.

The EOM are derived using Newtonian and Eulerian mechanics and are meant to be similar to the well recognized 6 DOF rigid body EOM for spacecraft. The translational and rotational motion are familiar equations with a few extra terms for the flexing behavior. The EOMs were developed to be efficient computationally. A one-way decoupling is introduced that allows for the center of mass acceleration, solar panel angular accelerations, and then the hub angular acceleration to be determined. Future work will investigate adding variable fuel tank mass and fuel slosh to this formulation.

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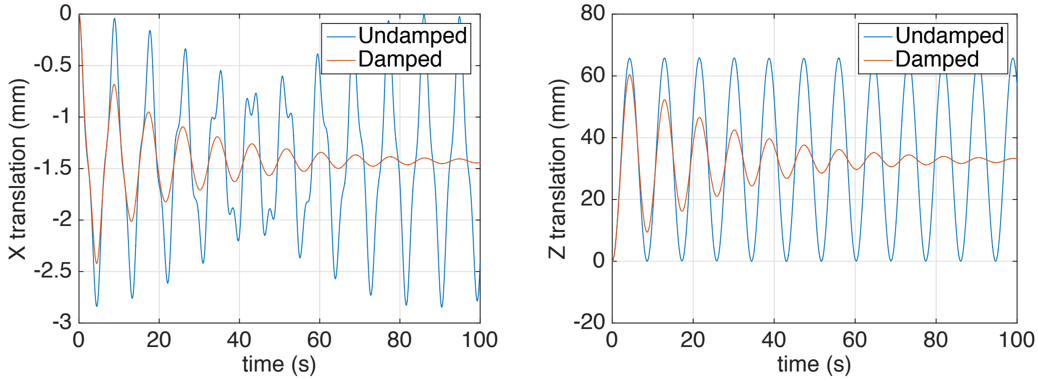
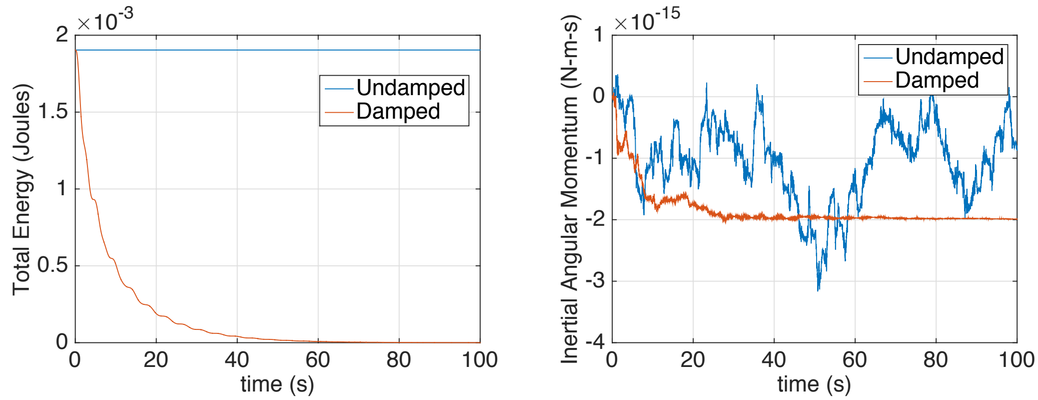


(a) X-direction translational motion (b) Z-direction translational motion

Figure 3. Translational motion of spacecraft



(a) Change of total energy of spacecraft (b) Total inertial angular momentum vector

Figure 4. Energy and momentum check

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